AN INTEGRATED VEHICLE ROUTING AND VEHICLE SEQUENCING PROBLEM AT THE CROSS-DOCKING CENTER: A META-HEURISTIC APPROACH

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The cross-docking is an innovative logistic strategy in the supply chain. The Cross-Docking Centre (*CDC*) functions as an intermediate destination. Cross-Docking Centres not only receive shipments from inbound vehicles but also dispatch them to outbound vehicles. To improve the efficiency of a supply chain, a literature review survey recommends to jointly considering operational level problems on cross-docking together. Therefore, in this study, the vehicle routing problem with moving shipments and vehicle sequencing problem at the *CDC* which has only single-receiving and single-shipping door are integrated and abbreviated as *VRSP-SD*. On the one hand, the integrated *VRSP-SD* differs from the literature by considering internal operations such as unloading, moving and reloading shipments not only at suppliers/ customers but also at *CDC*. On the other hand, the novelty of *VRSP-SD* in sequencing vehicles by minimizing the waiting time of vehicles based on the 'arrival time' of inbound vehicles to the *CDC* and also based on the 'route quantity' of outbound vehicles. Therefore, the objective of this study is to test the accuracy of the proposed Genetic Algorithm (*GA*) based meta-heuristic approach to solve *VRSP-SD* by optimizing the total transportation cost which incurred by routing vehicles, moving shipments and sequencing vehicles. A Mixed Integer Quadratic Programming (*MIQP*) model is developed to solve the integrated *VRSP-SD*. The developed mathematical *MIQP* model is coded in the *LINGO* (version 18) optimization software and Branch and Bound (*BB*) algorithm is used to obtain the exact optimal solution. The tuned parameters of the *GA* by the *Taguchi's* estimation scheme are used in the proposed approach. Moreover, the proposed *GA* is coded in the *MATLAB* platform to obtain a near optimal solution to the *VRSP-SD*. The data are extracted from the benchmark instances in the literature. Since the numerical experiments reveal the accuracy of the proposed *GA* as over 94% against the exact optimal solution obtain by *BB* algorithm, it is recommended to employ the proposed *GA* to solve the *VRSP-SD*. Therefore, the industries which apply cross-docking strategy may benefitted by utilizing this proposed *GA* to schedule the vehicles to the route as well as to the doors of CDC.

Keywords: cross-docking, meta-heuristic, vehicle routing, vehicle sequencing.

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INTRODUCTION

Cross-docking is an innovative logistic strategy in the supply chain. Also, the Cross-Docking Centre (CDC) is an intermediate destination in a supply chain network and it is exclusively dedicated to the transhipment of shipments. Therefore, these CDCs not only receive shipments from inbound vehicles but also dispatch them to outbound vehicles. Moreover, the doors at these CDCs are limited and most of the time they are less than the number of vehicles used to tranship the shipments. The 'vehicle routing problem' and 'vehicle sequencing problem' are the operational level decisions at the CDC. The literature survey (Van Belle et al., 2012) on the cross-docking recommends to jointly consider the operational level problems together. Consequently, integrating 'vehicle routing problem' and 'vehicle sequencing problem' will improve the efficiency of the supply chain. A CDC can have different layouts in its structure, but the basic layout has only a single-receiving door and a single-shipping door. Therefore, in the basic layout of CDC, scheduling vehicles to either the receiving door or the shipping door is a matter of sequencing vehicles to the door. This study is an extension of the research (Gnanapragasam & Daundasekera, 2022) on vehicle routing problem with moving shipments at the cross-docking centre (VRPCD-MS). Though a meta-heuristic method was utilized in the study (Gnanapragasam & Daundasekera, 2024) to the simplified version of the VRPCD-MS, sequencing vehicles problem is not integrated in it. In the current study, 'VRPCD-MS is integrated with the vehicle sequencing problem at the CDC which has single-door at either side' and abbreviated simply as VRSP-SD.

Since proper vehicle sequencing to the limited doors at the CDC optimizes the total transportation cost, (Yu & Egbelu, 2008) initiated the sequencing vehicles to the CDC with 'single-receiving door and single-shipping door' layout. The integrated model in this study differs from the literature in four ways. Firstly, the following internal operations in the consolidation process are taken into consideration: 'unloading shipments' at the receiving door, 'moving shipments' internally and 'reloading shipments' at the shipping door at the CDC. Secondly, 'loading/ unloading shipments' at the suppliers/customers are also taken into account. Thirdly, sequencing vehicles by minimizing the 'waiting time' of vehicles based on two different aspects. One aspect is based on the 'arrival time' of inbound vehicles to the CDC. The other aspect is based on the 'route quantity' of outbound vehicles. Finally, the development of the integrated VRSP-SD problem by including aforementioned three features. Therefore, the objective of this study is to test the accuracy of the proposed Genetic Algorithm based meta-heuristic approach to solve VRSP-SD in terms of total transportation cost incurred by routing vehicles, moving shipments and sequencing vehicles.

METHODOLOGY

A Mixed Integer Quadratic Programming (MIQP) model is developed to solve the integrated VRSP-SD. The single-objective function of the MIQP model is to minimize the total transportation cost which contains the following components: Cost of Travelling (*TC*) between suppliers/customers, Cost of Service (*SC*) at suppliers/ customers, Cost of Unloading (*UC*) shipments at receiving door, Cost of Moving (*MC*) shipments inside CDC, Cost of Loading (*LC*) at shipping door, Cost of vehicle Operations (*OC*), Cost of Changing over (*CC*) vehicle and Cost of Waiting (*WC*) vehicle.

Exact Optimal Solution to VRSP-SD by Branch and Bound (BB) Algorithm

Since VRPCD-MS is classified as a NP-hard problem (Gnanapragasam & Daundasekera, 2022), the integrated VRSP-SD is also a NP-hard problem. Moreover, Branch and Bound algorithm is capable of solving small-scale instances of NP-hard problems. Therefore, only the small-scale instances of the integrated VRSP-SD are considered to obtain the exact optimal solution. The developed mathematical MIQP model is coded in the *LINGO* (version 18) optimization software and Branch and Bound algorithm is used to obtain the exact optimal solution.

Near Optimal Solution to VRSP-SD by Genetic Algorithm (GA) based meta-heuristic approach

The flow chart in Figure 1 given below depicts all the necessary details about the proposed Genetic Algorithm based onmeta-heuristic approach:

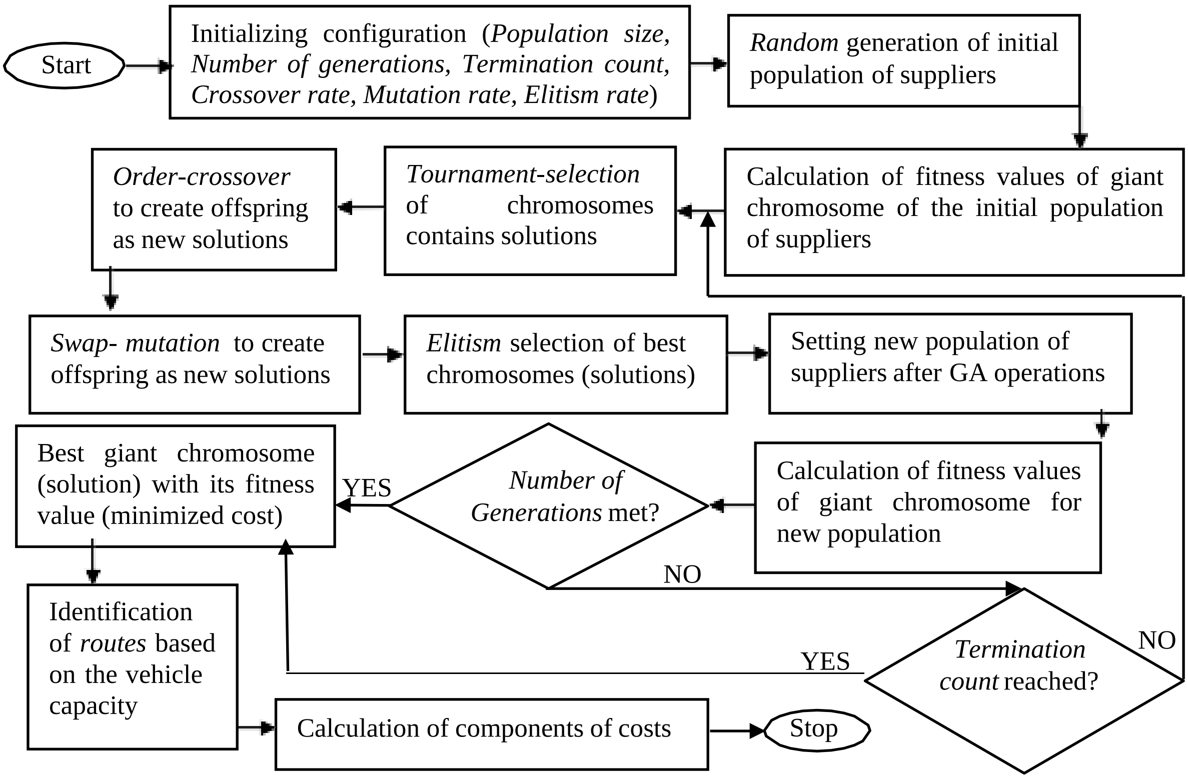

Figure 1: Flow chart of the Genetic Algorithm to solve the VRSP-SD

Initially the parameters of the Genetic Algorithm in Figure 1 are tuned by the *Taguchi's* estimation scheme. Accordingly, population size (*100*), number of generations (*150*), termination count (*50*), crossover rate (*0.7*), mutation rate (*0.3*), elitism rate (*0.1*) are estimated. Moreover, the proposed Genetic Algorithm is coded in the *MATLAB* platform to obtain a near optimal solution to the VRSP-SD. The data are extracted from the benchmark instances from (Wen et al., 2009) in the literature of Vehicle Routing Problem with Cross-Docking. The near optimal solution reached by Genetic Algorithm is compared with the exact optimal solution obtained by the Branch and Bound algorithm.

RESULTS AND DISCUSSION

The most prominent results of ten small-scale instances of VRSP-SD reached by both Branch and Bound (BB) algorithm and Genetic Algorithm (GA) are summarized in Table 1. In each instance, its size and required number of vehicles are included. In addition, exact optimal solution by the Branch and Bound algorithm, best solution and average solution of 10 repetitive execution ofthe Genetic Algorithm are also exhibited in Table 1. Since, the individual values of the cost components are not compared, only the total transportation costs obtained by both Branch and Bound and Genetic Algorithm approaches are presented in Table 1. Eventually, the Relative Percentage Deviation (RPD) value in Table 1 is calculated according to the following formula:

$$\mathrm{RPD}=\left[\frac{(\text{Average near OptimalSolution by GA approach})-(\text{Exact OptimalSolution by BBapproach})}{(\text{Exact OptimalSolution by BBapproach})}\right]\times 100$$

Table 1: Comparison of solutions between BB algorithm and GA for VRSP-SD

Instance Number	Instance Size		Required Vehicles		BB	GA		RPD in %
	Suppliers	Customers	Inbound	Outbound	Optimal Solution	Best Solution	Average Solution	
01	03	03	1	1	1,087.91	1,087.9	1,087.90	0.00
02	03	04	1	1	1,122.63	1,122.6	1,122.60	0.00
03	03	05	1	1	1,172.66	1,172.7	1,172.70	0.00
04	04	04	1	1	1,200.54	1,200.5	1,200.50	0.00
05	04	05	1	1	1,210.58	1,210.6	1,210.60	0.00
06	04	06	1	2	1,600.52	1,600.5	1,624.68	1.51
07	05	05	1	2	1,725.77	1,725.8	1,733.00	0.42
08	05	06	1	2	1,707.51	1,705.5	1,728.66	1.24
09	05	07	2	2	1,818.55	1,892.1	1,917.30	5.43
10	**06**	**06**	**2**	**2**	**1,981.20**	**2,009.2**	**2,052.48**	**3.60**

It can be observed from the results presented in the Table 1, that when the size of the instance is less than 10 (in the first 5 instances), the proposed Genetic Algorithm also reaches the same exact optimal solution obtained by the Branch and Bound algorithm. In the instances from 6th to 8th, although there is a small deviation (from 0.5% to 1.5% in RPD) in the average solution by GA, the best solution among the 10 replicates reaches the exact optimal solution. However, in the last two instances (9th and 10th), only a near optimal solution is reached by the proposed Genetic Algorithm with about 5.5% RPD. Therefore, it can be concluded that, the accuracy of the proposed Genetic Algorithm to solve VRSP-SD is over 94%. More details of the results of the best solution by Genetic Algorithm to the last instance (highlighted in bold in the Table 1) with 6-suppliers (labelled as S_1 to S_6) and 6-customers (labelled as C_1 to C_6) are summarized in Table 2 given below:

Table 2: Best solution by GA to the instance with 6-suppliers and 6-customers of VRSP-SD

Vehicle: Route	Route Quantity	Arrival/ Ready Time	Starting Time	Processing Time	Ending Time
1: CDC-S_6-S_5-CDC	18	147.01	147.01	33	180.01
2: CDC-S_2-S_4-S_3-S_1-CDC	42	212.08	212.08	57	269.08
3: CDC-C_2-C_6-C_3-CDC	22	306.08	306.08	37	343.08
4: CDC-C_5-C_1-C_4-CDC	38	**306.08**	**343.08**	53	396.08

Vehicle: Route	Components of total transportation cost								Route-wise cost
	TC	***SC***	***UC***	***MC***	***LC***	***OC***	***CC***	***WC***	
1: CDC-S_6-S_5-CDC	79.69	38	28	18	N/A	150	15	0	328.69
2: CDC-S_2-S_4-S_3-S_1-CDC	159.39	82	52	42	N/A	150	15	0	500.39
3: CDC-C_2-C_6-C_3-CDC	335.10	52	N/A	N/A	32	100	15	0	534.10
4: CDC-C_5-C_1-C_4-CDC	378.02	68	N/A	N/A	48	100	15	37	646.02
Total	**952.20**	**240**	**80**	**60**	**80**	**500**	**60**	**37**	**2009.20**

In the top part of the Table 2, the closed vehicle tours to collect shipments from suppliers and to distribute them to customers are respectively presented in the first two routes and the last two routes in the 1st-column. The accumulated shipments in each route are represented by the 'route quantity' in the 2nd-column. The 'arrival time' of inbound vehicles to the CDC, the 'ready time' of shipments to be loaded to outbound vehicles are included in the 3rd-column. The 'starting time' to process at the doors are added in the 4th-column. The 5th-column contains the 'processing time' of each vehicle. Finally at the last-column, the 'ending time' of each vehicleis mentioned. In the bottom part of the Table 2, the breakdowns of all the components of total transportation cost are reported from the 2nd-column to 9th-column. The final-column sums up the route-wise cost. Ultimately, the component-wise total costs are also highlighted at the last-row of the Table 2.

It can be observed from Table 2, that 1st-route vehicle arrives the CDC at 147.01 time units and starts to unload at its arrival. It finishes its job at 180.01 time units with 33 (which equals 18 *(accumulated units of shipments)* plus 15 *(vehicle changeover time)*) units of processing time. Also, the 2nd-route vehicle arrives much later 180.01 time units and that time the receiving door is not occupied by any vehicle. Therefore, without waiting, it starts its job at 212.08 time units. It stops its unloading at 269.08 time units with 57 units of processing time. Therefore, there is no waiting time for inbound vehicles. Moreover, it can be observed in Table 2, that the 'ready time' of shipments at the CDC is 306.08 but the 3rd-route vehicle has the least 'route quantity' and therefore it is assigned first to the shipping door. It finishes its loading at 343.08 time units with 37 units of processing time. However, the 4th-route vehicle has to wait until the 3rd-vehicle finishes its job and therefore at about 343.08 time units only it can start its loading. Hence, the waiting time of the 4th-vehicle is 37 (which is equal to 343.08 minus 306.08) time units. Finally, it completes its job at 396.08 time units with 53 units of processing time. In fact, only the 4th-route vehicle has to wait in this particular instance. Therefore, with 37 cost units of the waiting, the total transportation cost is 2009.20 cost units which is best solution of 10 execution of the last instance reported in the Table 1.

CONCLUSIONS AND RECOMMENDATION

The vehicle routing problem with moving shipments and vehicle sequencing problem at the cross-docking centre which has only a single-receiving and single- shipping door (VRSP-SD) are integrated in this study. The Genetic Algorithm (GA) based meta-heuristic approach is proposed to the mixed integer quadratic programming model developed to solve the integrated VRSP-SD. Since the accuracy of the proposed GA shows over 94% against the exact optimal solution obtains by Branch and Bound algorithm, it is recommended to employ the proposed GA to solve the VRSP-SD.Therefore, the industries which apply cross-docking strategy in their supply chain may benefitted by utilizing this proposed GA to schedule the vehicles to the route as well as to the doors of CDC.

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